Review

Knowing and Teaching Elementary Mathematics

Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States. (1999). Liping Ma. Lawrence Erlbaum Associates, xxv + 166 pp. ISBN 0-8058-2909-1(pb) $19.95. ISBN 0-8058-2908-3 (hb) $45.00.

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(The following review has been reprinted, with permission, from the *Notices of the American Mathematical Society,* Vol. 46, No. 8, pp. 881–887)

Notation: The reviewer will refer to the book under review as KTEM.

For all who are concerned with mathematics education (a set which should include nearly everyone receiving the *Notices*), KTEM is an important book. For those who are skeptical that mathematics education research can say much of value, it can serve as a counterexample. For those interested in improving precollege mathematics education in the U.S., it provides important clues to the nature of the problem. An added bonus is that, despite the somewhat forbidding educationese of its title, the book is quite readable. (You should be getting the idea that I recommend this book!)

Since the publication in 1989 of the *Curriculum and Evaluation Standards* by the National Council of Teachers of Mathematics [NCTM], there has been a steady increase in discussion and debate about reforming mathematics education in the U.S., including increased attention from university mathematicians (cf. [Ho]). Many mathematicians who take time to consider pre-college education form an intuition that it would help the situation if teachers knew more mathematics. If these mathematicians get more involved in mathematics education, they are likely to be surprised by how little this intuition seems to affect the agenda in mathematics education reform.

Partly this noninterest in mathematical expertise reflects an attitude widespread among educators [Hi] that "facts," and indeed all subject matter, are secondary in importance to a generalized, subject-independent teaching skill and the development of "higher-order thinking." Concerning mathematics in particular, the study [Be] is often cited as evidence for the irrelevance of subject matter knowledge. For this study, college mathematics training, as measured by courses taken, was used as a proxy for a teacher's mathematical knowledge. The correlation of this with student achievement was found to be slightly negative. A similar but less specific method was used in the recent huge Third International Mathematics and Science Study (TIMSS) of comparative mathematics achievement in forty-odd countries.

For TIMSS, U.S. students demonstrated adequate (in fourth grade) to poor (in twelfth grade) mathematics achievement [DoEd1–3]. To analyze whether teacher knowledge might help explain TIMSS outcomes, data on teacher training was gathered. In terms of college study, U.S. teachers appear to be comparable to their counterparts in other countries [DoEd1–3].

How can this intuition—that better grasp of mathematics would produce better teaching—appear to be so wrong? KTEM suggests an answer. It seems that successful completion of college course work is not evidence of thorough understanding of elementary mathematics. Most university mathematicians see much of advanced mathematics as a deepening and broadening, a refinement and clarification, an extension and fulfillment of elementary mathematics. However, it seems that it is possible to take and pass advanced courses without understanding how they illuminate more elementary material, particularly if one's understanding of that material is superficial. Over the past ten years or so, Deborah Ball and others [B1–3] have interviewed many teachers and prospective teachers, probing their grasp of the principles behind school mathematics. KTEM extends this work to a transnational context. The picture that emerges is highly instructive—and sobering. Mathematicians can be pleased to have at last powerful evidence that mathematical knowledge of teachers does play a vital role in mathematics learning. However, it seems also that the kind of knowledge that is needed is different from what most U.S. teacher preparation schemes provide, and we have currently hardly any institutional structures for fostering the appropriate kind of understanding.

The main body of KTEM (Chapters 1–4) presents the results of interviews with elementary school teachers from the U.S. (23 in all) and China (72 in all). The U.S. teachers were roughly evenly split between experienced teachers and beginners. Ma judged the group as a whole to be "above average." In particular, although "math anxiety" is rampant among elementary school teachers, this group had positive attitudes about mathematics: they overwhelmingly felt that they could handle basic mathematics and that they could learn advanced mathematics. The Chinese teachers were from schools chosen to represent the range of Chinese teaching experience and expertise: urban schools and rural, stronger schools and weaker.

The teachers' grasp of mathematics was probed in interviews organized around four questions. In summary form, the questions were as follows:

1) How would you teach subtraction of two-digit numbers when "borrowing" or "regrouping" is needed?

2) In a multiplication problem such as 123×645, how would you explain what is wrong to a student who performs the calculation as follows?

$$\begin{array}{r} 123 \\ \times \underline{645} \\ 615 \\ 492 \\ \underline{738} \\ 1845 \end{array}$$

(The student has correctly formed the partial products of 123 with the digits of 645, but has not "shifted them to the left," as required to get a correct answer.)

3) Compute $\frac{1\frac{3}{4}}{\frac{1}{2}}$. Then make up a story problem which models this computation, that is, for which this computation provides the answer.

4) Suppose you have been studying perimeter and area and a student comes to you excited by a new "theory": area increases with perimeter. As justification the student provides the example of a 4×4 square changing to a 4×8 rectangle: perimeter increases from 16 to 24, while area increases from 16 to 32. How would you respond to this student?

These questions are in order of increasing depth. The first two involve basic issues of place-value decimal notation. The third involves rational numbers and also involves division, the most difficult of the arithmetic operations. It further requires "modeling" or "representation"—connecting a calculation with a "real-world" situation. The last problem, which was originally stated in terms of perimeter and area of a "closed figure," potentially involves very deep issues. Even if one replaces "closed figure" with "rectangle," as all the teachers did, one must still compare the behavior of two functions of two real variables.

On sheepskin the American teachers seemed decidedly superior to the Chinese: they all were college graduates, and several had MAs. The Chinese teachers had nine years of regular schooling, and then three years of normal school for teachers—in terms of study time, a high school degree. However, measured in terms of mastery of elementary school mathematics, the Chinese teachers came out better.

The rough summary of the results of the interviews is: the Chinese teachers responded more or less as one would hope that a mathematics teacher would, while the American teachers revealed disturbing deficiencies. In more detail, on the first two problems, all teachers could perform the calculations correctly and could explain how to do them, that is, describe the correct procedure. However, even on the first problem, fewer than 20% of the U.S. teachers had a conceptual grasp of the regrouping process—decomposing one 10 into 10 ones. By contrast, the Chinese teachers overwhelmingly (86%) understood and could explain this decomposition procedure. On the second problem, about 40% of the U.S. teachers could explain the reason for the correct method of aligning the partial products, while over 90% of the Chinese teachers showed a firm grasp of the place value considerations that prescribe the alignment procedure.

On the third problem, a gap appeared even at the computational level: well under half of the American teachers performed the indicated calculation correctly. Only one came up with a technically acceptable story problem. Even this one was pedagogically questionable, since the units for the answer ($3\,{}^1\!/_2$) was persons, which children might expect to come in whole numbers. The Chinese teachers again all did the calculation correctly, and 90% of them could make up a correct story problem. Some suggested multiple problems, illustrating different interpretations of division.

On the fourth problem, the U.S. teachers did exhibit some good teaching instincts, and most, though not all, could state the formulas for area and perimeter of rectangles. However, when it came to analyzing the mathematics, they were lost at sea. Although most wanted to see more examples, over 90% were inclined to believe that the student's claim was valid. Some proposed to look something up in a book. Only three attempted a mathematical investigation of the claim, and again a lone one found a counterexample. The Chinese teachers also found this problem challenging, and most had to think about it for some time. After consideration, 70% of them arrived at a correct understanding, with valid counterexamples. Of the 30% who did not find the answer, most did think mathematically about the problem, though not sufficiently rigorously to find the defect in the student's proposal.

The contrast between the performances of the two groups of teachers was even more dramatic than this summary reveals. Some Chinese teachers gave responses that more than answered the question. They sometimes offered multiple solution methods. In the integer arithmetic problems, some indicated that, if the student was having trouble here, it meant that something more fundamental had not been learned properly. These comments point to a deeper layer of teaching culture that simply does not exist in the U.S. For example, American teaching of two-digit subtraction is usually based on "subtraction facts," the results of subtracting a one-digit number from a one- or two-digit number to get a one-digit number. These are simply to be learned by rote. The Chinese base subtraction on these same facts, but they refer to this topic as "subtraction within 20" and treat it as one to be understood thoroughly, since they regard it as the link between the computational and the conceptual basis for multidigit subtraction. In answering question 3, some Chinese teachers suggested that the given problem was too easy and offered harder ones. Also, the Chinese teachers were comfortable with the algebra that is implicitly involved in performing arithmetic with our standard decimal notation—for example, many explicitly invoked the distributive law when discussing multidigit multiplication. No such awareness of the algebraic backbone of arithmetic was shown by the American teachers.

In these first four chapters, KTEM also discusses issues of teaching methods. Without going into detail about this, I will report that the same limitations that teachers showed in giving a conventional explanation of a topic also prevented them from getting to the conceptual heart of the issue when using teaching aids such as manipulatives.

Thus, KTEM suggests that Chinese teachers have a much better grasp of the mathematics they teach than do American teachers. The hard-nosed might ask for evidence that this extra expertise actually produces better learning. Since Ma's work did not extend to a simultaneous study of the students of the teachers, KTEM cannot address this question. However, the substantial studies of Stevenson and Stigler [SS] do document superior mathematics achievement in China. (The Stevenson-Stigler project provided part of the motivation for Ma's work.) KTEM itself also provides some evidence of superior learning in China and of a sort directly related to the knowledge of teachers, as indicated in the interviews. The four interview questions were presented to a group of Chinese ninth-grade students from an unremarkable school in Shanghai. They all (with one quite minor lapse) could do all the calculations correctly and knew the perimeter and area formulas for rectangles. Over 60%

found a counterexample to the student's claim about area and perimeter, and over 40% could make up a story problem for the division of fractions in question 3. These Chinese ninth-grade students demonstrated better understanding of the interview problems than did the American teachers.

One should also entertain the possibility that Ma was overly optimistic in judging her group of American teachers to be "above average." However, this rating is broadly consistent with evidence from a much larger set of interviews conducted by Deborah Ball [B1–3] and also with the study [PHBL] of over two hundred teachers in the Midwest. In that study, for example, only slightly over half the subjects could provide an example of a number between 3.1 and 3.11. The portion of satisfactory responses to questions testing pedagogical competence was considerably smaller. The results of KTEM are also consistent with massive informal testimony from serious workers in professional development for teachers. The remarkable thing is that this problem—the failure of our system to produce teachers with strong subject matter knowledge and the negative impact of this failure—is not more explicitly recognized. Furthermore, solving this problem is not a major focus of mathematical education research and of education policy. I hope that KTEM will provide impetus for making it so.

KTEM gives us new perspectives on the problems involved in improving mathematics education in the U.S. For example, it strongly suggests that without a radical change in the state of mathematical preparedness of the American teaching corps, calls for teaching with or for "understanding," such as those contained in the NCTM *Standards,* are simply doomed. To the extent that they divert attention from the crucial factor of teacher preparedness, they may well be counterproductive. KTEM also indicates that claims that the traditional curriculum failed are misdirected. The traditional curriculum allowed millions of people to be taught reliable procedures for finding correct answers to important problems, without either the teachers or the students having to understand why the procedures worked. At the same time, students with high mathematical aptitude could learn substantially more mathematics, enough to support various technical or academic careers. This has to be counted a major success.

However, times have changed. The success of the traditional curriculum has fostered a mathematically based technology, which in turn has created conditions in which that curriculum is no longer appropriate. There are at least two reasons for this. First, we have cheap calculators that will do (at least approximately) any calculation of the elementary curriculum (and much more) with the push of a couple of buttons. These machines are typically much faster and more reliable than we are in doing these calculations. We also have "computer algebra" systems that will do more kinds of calculations than any single human knows how to do. It has always been one of the strengths of mathematics to seek reliable and systematic methods of computation, which has often meant creating algorithms. Anything that has been algorithmized can be done by a computer. Automation of calculation means that actually performing a calculation is no longer a problem working people usually have to worry about.

At the same time, it means that calculation is much more prevalent than before. Hence, people have to spend more time determining what calculation to do. That is the second reason that mathematics education needs to change. My daughter was a solid mathematics student but had no enthusiasm for the subject and did not expect to use it in whatever career she might choose. Now she works in management consulting, and she finds that her high school algebra comes in handy in creating spreadsheets. Simply learning computational procedures without understanding them will not develop the ability to reason about what sort of calculations are needed. In short, to function at work, people now need more understanding and less procedural virtuosity than they did a generation ago. (Who knows what they will need in another generation!)

The good news from KTEM is that there is no serious conflict between procedural knowledge and conceptual knowledge: Chinese teachers seem to be able to develop both in their students. (This is another intuition of most mathematicians I know who have been studying educational issues: it should be the case that procedural ability and conceptual understanding support each other. The Chinese teachers had a traditional saying to describe this learning goal: "Know how, and also know why.") The bad news is that our current teaching corps is not capable of delivering this kind of double understanding: we can only reasonably ask them for procedural facility. Let us be clear that this is not a matter of teachers lacking certification or teaching outside their specialty, which are both frequent problems that aggravate the situation. The certification procedures, the teaching methods courses, most college mathematics courses, the recruitment processes, the conditions of employment, most current teacher development—none of these is geared to ensuring that U.S. mathematics teachers have themselves the understanding needed to teach for understanding. In short, virtually the whole American K–12 mathematics education enterprise is out of date.

How might the U.S. create a teaching corps with capabilities more like those of the Chinese teachers? To begin to answer, we should try to be precise as to what the differences are between the two groups. From the evidence of KTEM, I would list three salient differences:

1. Chinese teachers receive better early training—good training produces good trainers, in a virtuous cycle.

2. Chinese mathematics teachers are specialists. Making mathematics teaching a specialty can be expected to increase the mathematical aptitude of the teaching corps in two ways: it reduces the manpower requirements for mathematics education by concentrating it in the hands of the mathematically most qualified teachers, and it raises the incentives for mathematically inclined people to become teachers. Beyond its recruitment implications, it means that Chinese teachers have more time and motivation for developing their understanding of mathematics. This self-improvement is amplified by a social effect: specialization creates a corps of colleagues who can work together to deepen the common teaching culture in mathematics. Thus, making mathematics teaching a specialty works in multiple ways to increase the quality of mathematics education.

3. Chinese teachers have working conditions which favor maturation of understanding. U.S. teachers spend virtually their whole day in front of a class, while the Chinese teachers have time during the school day to study their teaching materials, to work with students who need or merit special attention, and to interact with colleagues. New teachers can learn from more experienced ones. All can study together the key aspects of individual lessons, an activity they engage in systematically. They can also sharpen their skills by discussing mathematical problems. Stevenson and Stigler [SS] have observed that time for self-development is a general feature of mathematics education in East Asia, which, to go by TIMSS [DoEd1–3] as well as [SS], has the most successful systems of mathematics education in the world today.

The combination of training, recruitment, and job conditions that prevails in China helps produce a level of teaching excellence that Ma calls PUFM, “profound understanding of fundamental mathematics.” PUFM and how it is attained is the concern of Chapters 5 and 6. It is important to understand that PUFM involves more than subject matter expertise, vital as that is; it also involves how to communicate that subject matter to students. Education involves two fundamental ingredients: subject matter and students. Teaching is the art of getting the students to learn the subject matter. Doing this successfully requires excellent understanding of both. As simple and obvious as this proposition may seem, it is often forgotten in discussions of mathematics education in the U.S., and one of the two core ingredients is emphasized over the other. In K–12 education the tendency is to emphasize knowing students over knowing subject matter, while at the university level the emphasis is frequently the opposite. (This cultural difference may well be part of the reason some university mathematicians have reacted negatively to the NCTM *Standards*. The emphasis on teaching methods over subject matter is prominent in the recommendations and “vignettes” of this document.) Both these views of teaching are incomplete.

What educational policies in the U.S might promote the development of a teaching corps in which PUFM were, if not commonplace, at least not extremely rare? This question is discussed in Chapter 7, the final chapter of KTEM. I would like to add my own perspective on the issue. The differences (1), (2), and (3) listed above suggest part of the answer.

Differences (2) and (3) are primarily matters of educational policy. No revolution in American habits is required to create mathematics specialists or to give them opportunity for study and collegial interaction. What is mainly required is political will.

Regarding difference (2), the manpower considerations which favor mathematics specialists beginning in the early grades are much stronger in the U.S. than in China. The U.S information society has much higher demand for mathematically able people than does the predominantly rural economy of China. Hence, schools face much heavier competition for mathematically competent personnel, and every policy that could lower their manpower requirements or improve their competitive position would benefit mathematics education. The difference in technological level also makes the need for coherent mathematics education greater in the U.S. than in China. Simply partitioning the present cadre of elementary teachers into math specialists and nonmath would already offer the average child a better-qualified (elementary) math teacher while relieving many others of what is now an onerous duty, all without raising overall personnel requirements. Some educators have for some time been calling for mathematics specialists even in the elementary grades [Us]. Perhaps the evidence from KTEM that having teachers who understand mathematics can make a difference already in the second grade (the usual time for two-digit subtraction) can convince education policymakers to heed this call.

Regarding difference (3), testimony from interviews of teachers with PUFM indicates that having time for study and collegial interaction is an important factor in developing PUFM. Such time would be most productive in the context of mathematics specialists—both study and discussion would be more focused on mathematics. Scheduling this time might be more controversial than creating specialists because it requires resources. In fact, in East Asia classes are larger than here, so a given teacher there handles about the same number of students as does a teacher in the U.S.[SS]. The improvement in lessons promoted by study and interaction with colleagues seems to more than make up for larger class size. There is currently in the U.S. a call to reduce class size. On the evidence of KTEM and [SS], I believe that the resources required for such a change would be better spent in eliminating difference (3).

What will be hardest is eliminating difference (1), that is, establishing in the U.S. the virtuous cycle, in which students would already graduate from ninth grade or from high school with a solid conceptual understanding of mathematics, a strong base on which to build teaching excellence. I expect that movement in that direction will, at least at the start, require massive intervention from higher education. New professional development programs, both preservice and in-service, that focus sharply on fostering deep understanding of elementary mathematics in a teaching context will need to be created on a large scale. Current university mathematics courses will not serve; as KTEM makes clear, the needs of teachers at present are of a completely different nature from the needs of professional mathematicians or technical users of mathematics, for whom almost all current offerings were designed.

I would recommend that these programs be joint efforts of education departments and mathematics departments to guarantee that the two poles of teaching, the subject matter and the pedagogy, both get emphasized. These departments have rather different cultures, and developing productive working relationships will not be a simple task; but with sufficient backing from policymakers who understand the current purposes and needs of mathematics education and the shortfall between current capabilities and these needs, some beneficial programs should emerge.

While the greatest need for improvement is probably at the elementary level, middle school and secondary teachers should not be neglected in the new professional development programs. Undoubtedly they know more mathematics than the typical elementary school teacher, but they too must have suffered from the lack of attention to understanding during their early education. Moreover, they need to deal with a larger body of material than do elementary teachers.

There is also the issue of texts. The Chinese teachers have materials, texts, and teaching guides that support their self-study. American texts tend to be lavishly produced but disjointed in presentation [Sc, DoEd1-3], and the teacher's guides do not help much either. Thus, the intervention programs should also work to create materials which will help teachers both learn and transmit a coherent view of mathematics. Eventually, these might be the basis for new texts.

At least at the start, these programs should be multiyear in scope, both so that teachers who do not have the favorable working conditions of Chinese teachers can nevertheless refresh and progressively improve their understanding of mathematics and so that those teachers who do obtain such working conditions can get to the level where self-directed study can be a reliable mode of improvement. One of the most outmoded ideas in education is that a teacher can reasonably be expected to know all that he or she needs to know, of subject matter or teaching, at the start of work. Continued study, especially of subject matter, since teaching itself will provide plenty of opportunities for learning about children, should become the norm. If a program of this sort is implemented successfully, it should gradually become less necessary. The step-by-step improvement in education provided by teachers with better understanding and the gradual deepening of teaching culture by teachers interacting collegially among themselves should allow elaborate development programs to shrink and eventually disappear or to shift to study of more sophisticated topics, becoming, in subject matter at least, more like standard college mathematics courses. This would constitute truly satisfying progress in our system of mathematics education. However, it will require great effort and resolve to achieve.

In summary, KTEM has lessons for all educational policymakers. Legislators, departments of education, and school boards need to understand the potential value in creating a corps of elementary-grade mathematics specialists who have scheduled time for study and collegial interaction. University educators need to understand teacher training in mathematics as a distinct activity, different from but of comparable value to training scientists, engineers, or generalist teachers. I believe that these mutually supportive changes would give us a fighting chance for successful mathematics education reform.

Getting the Mathematics to the Students

Ma's notion of "profound understanding of fundamental mathematics (PUFM)," involves both expertise in mathematics and an understanding of how to communicate with students. Teacher Mao, one of the teachers Ma identified as possessing PUFM, eloquently expressed the need for both types of understanding:

I always spend more time on preparing a class than on teaching, sometimes three, even four times the latter. I spend the time in studying the teaching materials; what is it that I am going to teach in this lesson? How should I introduce the topic? What concepts or skills have the students learned that I should draw on? Is it a key piece on which other pieces of knowledge will build, or is it built on other knowledge? If it is a key piece of knowledge, how can I teach it so students grasp it solidly enough to support their later learning? If it is not a key piece, what is the concept or the procedure it is built on? How am I going to pull out that knowledge and make sure my students are aware of it and the relation between the old knowledge and the new topic? What kind of review will my students need? How should I present the topic step-by-step? How will students respond after I raise a certain question? Where should I explain it at length, and where should I leave it to students to learn it by themselves? What are the topics that the students will learn which are built directly or indirectly on this topic? How can my lesson set a basis for their learning of the next topic, and for related topics that they will learn in their future? What do I expect the advanced students to learn from this lesson? What do I expect the slow students to learn? How can I reach these goals? etc. In a word, one thing is to study whom you are teaching, the other thing is to study the knowledge you are teaching. If you can interweave the two things together nicely, you will succeed. We think about these two things over and over in studying teaching materials. Believe me, it seems to be simple when I talk about it, but when you really do it, it is very complicated, subtle, and takes a lot of time. It is easy to be an elementary school teacher, but it is difficult to be a good elementary school teacher.

I would like to highlight the concern in Teacher Wang's statement for the connectedness of mathematics, the desire to make sure that students see mathematics as a coherent whole. This is certainly how mathematicians see it, and to us it is one of the major attractions of the field: mathematics makes sense and helps us make sense of the world. For me, perhaps the most discouraging aspect of working on K–12 educational issues has been confronting the fact that most Americans see mathematics as an arbitrary set of rules with no relation to one another or to other parts of life. Many teachers share this view. A teacher who is blind to the coherence of mathematics cannot help students see it.

—R. H.

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